

(03 HOURS)

(MAX. MARKS : 80)

Note:

1. Question No. 1 is compulsory.
2. Attempt **any three** questions out of remaining **five** questions.
3. Assume suitable data wherever necessary.
4. Figures to right indicate full marks.

- Q.1** Answer the following (Any four) **Marks**
- a. If $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ Find the characteristic roots of $A^3 + I$. **05**
- b. Evaluate $\int f(z) dz$ along the parabola $y = 2x^2$ from $z = 0$ to $z = 3 + 18i$ Where $f(z) = x^2 - 2iy$. **05**
- c. Determine all basic solutions and optimal basic feasible solution to the following problem. **05**
 Max. $z = x_1 + 3x_2 + 3x_3$
 Subject to $x_1 + 2x_2 + 3x_3 = 4$,
 $2x_1 + 3x_2 + 5x_3 = 7$,
 $x_1, x_2, x_3 \geq 0$.
- d. Find the z-transform of $f(k) = 3^k, k \geq 0$. **05**
- Q.2** a. Find the Eigenvalues and Eigenvectors of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$. **06**
- b. The heights of six randomly chosen sailors are in inches: 63, 65, 68, 69, 71 and 72. The heights of ten randomly chosen soldiers are: 61, 62, 65, 66, 69, 69, 70, 71, 72, and 73. discuss in the light that these data throw on the suggestion that the soldiers on an average are taller than sailors. **06**
- c. Use the dual simplex method to solve the L.P.P. **08**
 Maximize $z = -3x_1 - 2x_2$
 Subject to $x_1 + x_2 \geq 1$;
 $x_1 + x_2 \leq 7$;
 $x_1 + 2x_2 \geq 10$;
 $0x_1 + x_2 \leq 3$;
 $x_1, x_2 \geq 0$
- Q.3** a. Find the relative maximum or minimum of the function **06**
 $Z = x_1^2 + x_2^2 + x_3^2 - 8x_1 - 10x_2 - 12x_3 + 100$.
- b. If $f(k) = 4^k U(k)$ and $g(k) = 5^k U(k)$, then find the Z-transform of $\{f(k) * g(k)\}$. **06**
- c. Find all possible Laurents expansion of $f(z) = \frac{z}{(z-1)(z-2)}$ about $z = -2$. **08**

Q.4 a. Verify Cayley-Hamilton theorem for the matrix A and hence find the matrix represented by 06

$$\text{by } A^6 - 6A^5 + 9A^4 + 4A^3 - 12A^2 + 2A - I \text{ where } A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$$

b. In a survey of 200 boys of which 75 were intelligent, 40 had educated fathers, while 90 of the unintelligent boys had uneducated fathers. Do these figures support the hypothesis that educated fathers have intelligent boys. 06

c. Using the Kuhn-Tucker conditions to solve the N.L.P.P 08

$$\text{Maximize } z = 8x_1 + 10x_2 - x_1^2 - x_2^2$$

$$\text{Subject to } 3x_1 + 2x_2 \leq 6;$$

$$x_1, x_2 \geq 0$$

Q.5 a. Evaluate $\oint \frac{3z^2+z}{z^2-1} dz$ using Cauchy's residue theorem, 06

where C is the circle $|z| = 2$.

b. Using the method of Lagrange's multiplier solve the N.L.P. 06

$$\text{Optimize } z = 10x_1 + 8x_2 + 6x_3 + 2x_1^2 + x_2^2 + 3x_3^2 - 100.$$

$$\text{Subject to } x_1 + x_2 + x_3 = 20.$$

$$x_1, x_2, x_3 \geq 0.$$

c. The marks obtained by 1000 students in an examination are found to be normally 08

Distributed with mean 70 and s. d. 5. Estimate the number of students whose marks will be (i) between 60 and 75 (ii) more than 75.

Q.6 a. Find the inverse z-transform of $F(z) = \frac{1}{(z-3)(z-2)}$ if ROC is $2 < |z| < 3$. 06

b. Show that the matrix $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ is diagonalisable. find the diagonal form D and diagonalizing matrix M. 06

c. Solve the L.P.P by simplex method. 08

$$\text{Maximize } z = 4x_1 + 3x_2 + 6x_3$$

$$\text{Subject to } 2x_1 + 3x_2 + 2x_3 \leq 440;$$

$$4x_1 + 0x_2 + 3x_3 \leq 470;$$

$$2x_1 + 5x_2 + 0x_3 \leq 430;$$

$$x_1, x_2, x_3 \geq 0.$$